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Wave Propagation in Rectangular Nanoplates Based on Strain Gradient Elasticity Theory and Considering in-Plane Magnetic Field

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Abstract: Through strain gradient elasticity theory, this paper has studies wave propagation in rectangular nanoplates by considering in-plane magnetic field. The strain gradient theory has two gradient parameters and has the ability to be compared with the nonlocal elasticity theory. To the best of the authors' knowledge, this theory has been used for the first time in order to investigate the wave propagation in nanoplates. Moreover, It is the first time that magnetic field has been considered in modeling the wave propagation in rectangular nanoplates. An analytical method is adopted to achieve a closed-form solution for the governing equation. To verify the present methodology, the results are compared with others' findings. It is obtained that with the increase in static gradient parameter, the frequencies have been increased. Furthermore, it is shown that the phase velocities increase for the increase of magnetic field.

Keywords: Aifantis's strain gradient elasticity theory, Wave propagation, Magnetic field, Rectangular nanoplates

1. Introduction

Before using any mechanical process such as material forming to form an object, it may be important to find the material properties. For this purpose, several methods are available in the literature. For an example, nondestructive methods such as studying the wave propagation in objects can be used for finding material properties. In the present paper, the wave propagation of nanoplates in magnetic field is investigated which may be useful for predicting their properties. nano structures have been studied up to now, considering magnetic field for several times. Kiani [1] studied the free in-plane and out-of-plane vibration behaviors of conducting rectangular nanoplates subjected to unidirectional in-plane steady magnetic fields. The body forces exerted on the nanoplate based on the hypotheses of Kirchhoff, Mindlin, and higher-order plate theories were obtained. Leng et al. [2] investigated the thermal stability and magnetic anisotropies of nickel nanoplates with {111} planes as the exposure plane. It was shown that as the angle between the film plane and the applied field direction varies from zero to 45° and to 90°, the coercivity measured at 5 K increases from 335 Oe to 373 and to 410 Oe. Klinovaja et al. [3] derived an effective low-energy theory for metallic (armchair and non-armchair) single-wall nanotubes in the presence of an electric field perpendicular to the nanotube axis, and in the presence of magnetic fields, taking into account spin-orbit interactions. Kono et al. [4] reviewed recent progress in the theoretical and experimental studies of single-wall carbon nanotubes in high magnetic fields. Low-temperature magneto-transport experiments demonstrated the influence of quantum interference, disorder, and band-structure effects. Ghorbanpour Arani et al. [5] presented the effect of longitudinal magnetic field on wave propagation of an embedded double-walled carbon nanotube with conveying fluid using either the Euler–Bernoulli beam or the Timoshenko beam models. Conveying fluid is magnetite (Fe_3O_4) nanofluid, which

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was a ferrofluid. Metlov and Guslienko [6] studied the stability of magnetic vortex with respect to displacement of its center in a nano-scale circular cylinder made of soft ferromagnetic material theoretically. The mode of vortex displacement producing no magnetic charges on the cylinder side was proposed and the corresponding absolute single-domain radius of the cylinder was calculated. Li et al. [7] reported the results of an investigation into the effect of transverse magnetic fields on dynamic characteristics of multi-walled carbon nanotubes (MWNTs). Couple dynamic equations of MWNTs subjected to a transverse magnetic field were derived and solved by considering the Lorentz magnetic forces induced by a transverse magnetic field exerted on MWCNTs. Homogeneous dispersion of multi-walled carbon nanotubes (CNTs) in aqueous solution was achieved using several dispersants by Jang et al. [8]. The most efficacious dispersant was Polyethyleneimine (PEI). Stable CNT dispersions were found to have higher zeta potentials compared to poorly dispersed suspensions. Murmu et al. [9] investigated the effect of an in-plane magnetic field on the vibration of a magnetically sensitive single-layer graphene sheet using equivalent nonlocal elastic plate theory. Governing equations for nonlocal vibration of the SLGS under an in-plane magnetic field were derived considering the Lorentz magnetic force obtained from Maxwell’s relation. This paper has disscused, an analytical solution for the wave propagation in rectangular nanoplates with considering in-plane magnetic field based on a new strain gradient elasticity theory with two gradient parameters. To the best of the authors' knowledge, it is the first time that this gradient theory has been used in studying wave propagation in nanoplates. It is also the first time that magnetic field is considered for investigating wave propagation in rectangular nanoplates. The effects of different parameters such as gradient parameters are studied on the frequencies, phase velocities and group velocities.

2. Strain Gradient Theory with Two Parameters

Various formats of gradient elasticity are used in the studies of nano structures [10-11]. According to the strain gradient elasticity theory with two gradient parameters reported by Askes and Aifantis [11], following stress-strain relations are used,

$$\left(\sigma_{ij} - \mu\sigma_{ij,mm}\right) = C_{ijkl}\left(\varepsilon_{ij} - l\varepsilon_{ij,mm}\right) \tag{1}$$

where σ_{ij} and ε_{ij} are stress and strain tensors and C_{ijkl} are elastic constants, while l and μ are defined as static and dynamic gradient parameters, respectively. For static analysis, μ can be equal to zero and the above constitutive equation may be the same as Papargyri-Beskou and Beskos [12]. A difference between nonlocal elasticity theory and Aifantis’s gradient theory concerns how the balance of momentum is formulated: Eringen uses the divergence of σ_{ij} whereas the Aifantis’s strain gradient elasticity theory uses the divergence of the right-hand side of Eq. (1).

3. Governing Equations

Considering the size effects is the main difference between different theories used for nanoplates and those adopted for macro plates. In investigating nanoplates, instead of using the Hook’s law, other stress-strain relations with considering small scale effects should be used. One of these stress-strain relations was defined in Eq. (1). By substituting strain-displacements in this equation, one can have three following equations,

$$\sigma_x - \mu\nabla^2\sigma_x = \frac{E}{1-v^2}\left(-z\frac{\partial^2 w}{\partial x^2} - vz\frac{\partial^2 w}{\partial y^2}\right) - \frac{El}{1-v^2}\nabla^2\left(-z\frac{\partial^2 w}{\partial x^2} - vz\frac{\partial^2 w}{\partial y^2}\right) \tag{2}$$

$$\sigma_x - \mu\nabla^2\sigma_x = \sigma_y - \mu\nabla^2\sigma_y = \frac{E}{1-v^2}\left(-z\frac{\partial^2 w}{\partial y^2} - vz\frac{\partial^2 w}{\partial x^2}\right) - \frac{El}{1-v^2}\nabla^2\left(-z\frac{\partial^2 w}{\partial y^2} - vz\frac{\partial^2 w}{\partial x^2}\right) \quad (3)$$

$$\tau_{xy} - \mu\nabla^2\tau_{xy} = \frac{E}{2(1+v)}\left(-2z\frac{\partial^2 w}{\partial x\partial y}\right) - \frac{El}{2(1+v)}\nabla^2\left(-2z\frac{\partial^2 w}{\partial x\partial y}\right) \quad (4)$$

Similar to the definition of resultant moments for macro plates, following relations can be achieved by differentiating and integrating from Eqs. (2-4),

$$\frac{\partial^2 M_x}{\partial x^2} - \mu\nabla^2(\frac{\partial^2 M_x}{\partial x^2}) = \frac{Eh^3}{12(1-v^2)}\left(-\frac{\partial^4 w}{\partial x^4} - v\frac{\partial^4 w}{\partial x^2\partial y^2}\right) - \frac{Eh^3 l}{12(1-v^2)}(-\frac{\partial^6 w}{\partial x^6} - \frac{\partial^6 w}{\partial x^4\partial y^2} - v\frac{\partial^6 w}{\partial x^2\partial y^4} - v\frac{\partial^6 w}{\partial x^4\partial y^2}) \quad (5)$$

$$\frac{\partial^2 M_y}{\partial y^2} - \mu\nabla^2(\frac{\partial^2 M_x}{\partial x^2}) = \frac{Eh^3}{12(1-v^2)}\left(-\frac{\partial^4 w}{\partial y^4} - v\frac{\partial^4 w}{\partial x^2\partial y^2}\right) - \frac{Eh^3 l}{12(1-v^2)}(-\frac{\partial^6 w}{\partial y^6} - \frac{\partial^6 w}{\partial y^4\partial x^2} - v\frac{\partial^6 w}{\partial y^2\partial x^4} - v\frac{\partial^6 w}{\partial y^4\partial x^2}) \quad (6)$$

$$\frac{\partial^2 M_{xy}}{\partial x\partial y} - \mu\nabla^2(\frac{\partial^2 M_x}{\partial x^2}) = \frac{Eh^3}{12(1+v)}\left(\frac{\partial^4 w}{\partial x^2\partial y^2}\right) - \frac{Eh^3 l}{12(1+v)}\left(\frac{\partial^6 w}{\partial y^4\partial x^2} + \frac{\partial^6 w}{\partial y^2\partial x^4}\right) \quad (7)$$

Where $(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2}(\sigma_x, \sigma_y, \tau_{xy})zdz$. In deriving the equilibrium equation for nanoplates, there is no difference between macro and nano plates. So the equilibrium equation on the basis of Kirchhoff plate theory can be expressed as follow,

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} + q(x,y) = \rho h\frac{\partial^2 w}{\partial t^2} \quad (8)$$

Now by adding Eqs. (5-6) and inserting Eq. (8) in it, following governing equation with static and dynamic gradient parameters for wave propagation in nanoplates will be achieved,

$$-q(x,y) + \rho h\frac{\partial^2 w}{\partial t^2} - \mu\nabla^2\left(-q(x,y) + \rho h\frac{\partial^2 w}{\partial t^2}\right) = D\left(-\frac{\partial^4 w}{\partial x^4} - \frac{\partial^4 w}{\partial y^4} - 2\frac{\partial^4 w}{\partial x^2\partial y^2}\right) - lD\left(-\frac{\partial^6 w}{\partial x^6} - \frac{\partial^6 w}{\partial y^6} - 3\left(\frac{\partial^6 w}{\partial x^4\partial y^2} + \frac{\partial^6 w}{\partial x^2\partial y^4}\right)\right) \quad (9)$$

Where $= \dfrac{Eh^3}{12(1-v^2)}$. In order to introduce the magnetic field in this paper, first we explain the Maxwell's theory briefly. The main five Maxwell's relations are defined as below [13],

$$\begin{aligned}
&\boldsymbol{J} = \nabla\times\boldsymbol{h}\\
&\nabla\times\boldsymbol{e} = -\eta\frac{\partial\boldsymbol{h}}{\partial t}\\
&\nabla.\boldsymbol{h} = 0\\
&\boldsymbol{e} = -\eta(\frac{\partial\boldsymbol{U}}{\partial t}\times\boldsymbol{H})\\
&\boldsymbol{h} = \nabla\times(\boldsymbol{U}\times\boldsymbol{H})
\end{aligned} \quad (10)$$

Where η is the magnetic field permeability, $\boldsymbol{U}$ is the displacement field, $\boldsymbol{J}$ is the current density, $\boldsymbol{e}$ is the strength vector and $\boldsymbol{h}$ is the distributing vector. In this article, it is assumed that the in-plane magnetic field is $\boldsymbol{H} = (H_x, 0,0)$. Hence, the distributing vector and the current density are defined as below [13],

$$\boldsymbol{h} = \nabla\times(\boldsymbol{U}\times\boldsymbol{H}) = -H_x\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)i + H_x\frac{\partial v}{\partial x}j + \frac{\partial w}{\partial x}k$$

$$\boldsymbol{J} = \nabla\times\boldsymbol{h} = H_x\left(\frac{\partial^2 v}{\partial x\partial z} + \frac{\partial^2 w}{\partial x\partial y}\right)i - H_x\left(\frac{\partial^2 v}{\partial y\partial z} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right)j + H_x\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y\partial z}\right)k \quad (11)$$

Therefore the Lorentz force in Cartesian coordinates can be expressed as [13],

$$f_x = 0$$

$$f_y = \eta H_x^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} \right)$$

$$f_z = \eta H_x^2 \left(\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \tag{12}$$

Now by substituting the magnetic force ($q = \int_{-\frac{h}{2}}^{\frac{h}{2}} f_z dz$) in governing Eq. (9), one can investigate the influences of magnetic field on nanoplates. To study the wave propagation in rectangular nanoplates including two gradient parameters, following distribution for transverse displacement can be assumed [14],

$$w = We^{i(xk_x + yk_y - \omega t)} \tag{13}$$

where k_x and k_y are the wave numbers in the *x* and *y* directions, respectively. This assumption is used for investigating bulk waves in nanoplates which are independent of boundary conditions. This type of bulk waves may be used as a method for nondestructive testing. By substituting above approximation in the governing Eq. (9), following relation can be found as,

$$\omega = \sqrt{\frac{D\left(k_x^4+k_y^4+2k_x^2k_y^2\right)+lD\left(k_x^6+k_y^6+3(k_x^4k_y^2+k_x^2k_y^4)\right)+\eta hH_x^2\left(k_x^2+k_y^2\right)+\mu\eta hH_x^2(k_x^4+k_y^4+2k_x^2k_y^2)}{\rho h+\mu\rho h(k_x^2+k_y^2)}} \tag{14}$$

4. Numerical Results

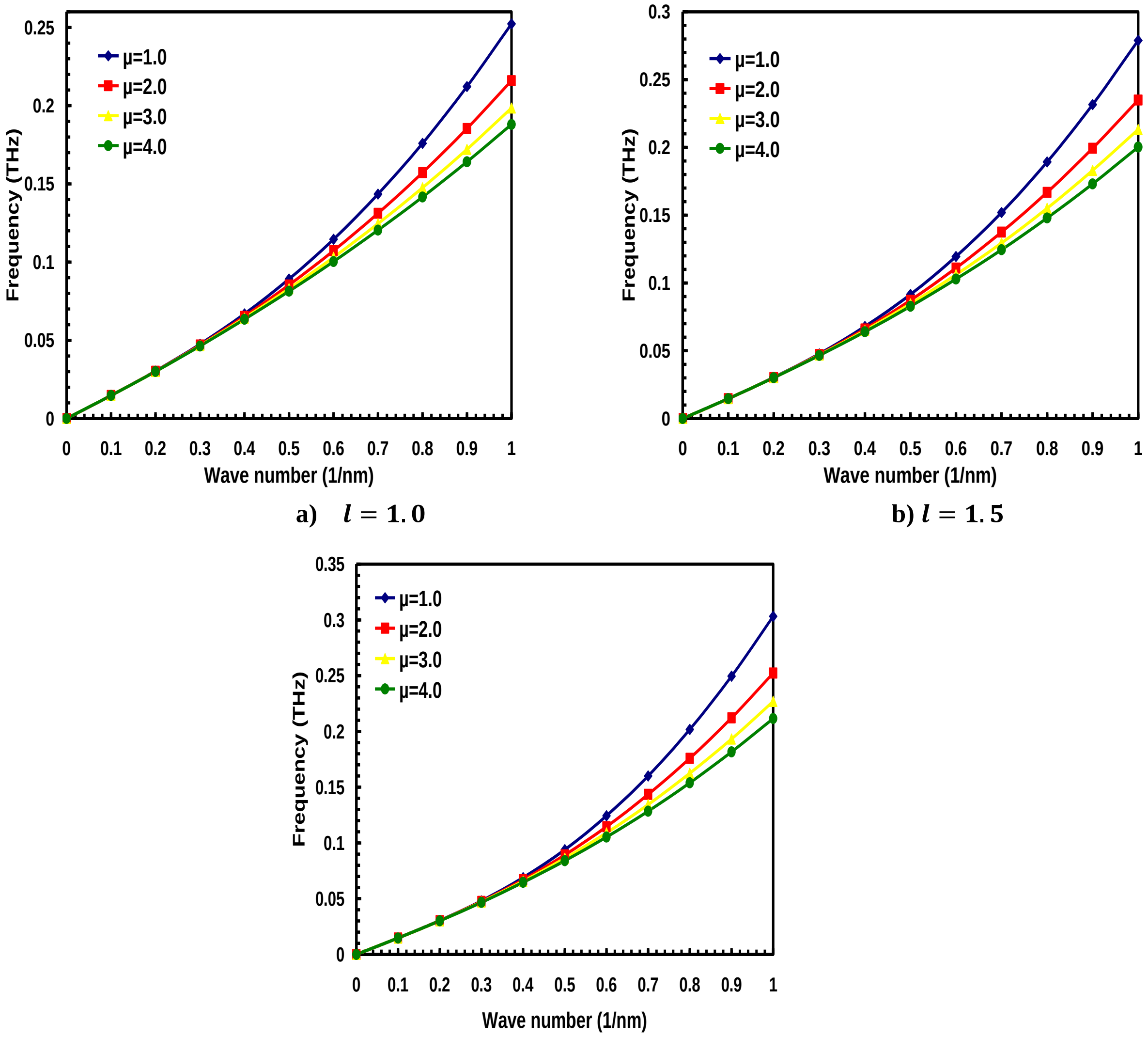
The formulation developed based on the strain gradient elasticity theory with the two gradient parameters will be employed to conduct a quantitative analysis on the wave propagation in rectangular nanoplates with considering magnetic field. Since there are no published results available for wave propagation in rectangular nanoplates on the basis of Aifantis's gradient theory in open literature, the results of nonlocal elasticity theory are used for comparison. In Table 1, our results are compared with the results of Wang et al. [14] for different nonlocal parameter. It was seen that our numerical results are in a good agreement with the results of nonlocal elasticity theory. It may be important to note that although the results in Table 1 do not show any advantage for the Aifantis strain gradient theory, other results in the literature may show this easily [11]. Moreover, the results in Fig. 1 can show the importance of this gradient theory in comparison with nonlocal theory, aswell. Moreover, results of the present work can be compared with the outcomes of a paper done by the present authors [15].

Table 1. A comparison between the results of the paper and those reported by nonlocal elasticity theory ($k_x = k_y = 1 \times 10^8$).

	$\mu(nm^2)$								
$f(THz)$	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
Present	0.002055	0.002052	0.002048	0.002044	0.00204	0.002036	0.0020316	0.002028	0.002024
[14]	0.002055	0.002052	0.002048	0.002044	0.00204	0.002036	0.0020316	0.002028	0.002024

In Fig. 1, the dispersion curves for rectangular nanoplates are presented. The effects of static and dynamic gradient parameters are also investigated. It is observed that with the increase of wave numbers, the frequencies will increase. In addition, it is shown that increasing the dynamic gradient parameter will decrease the frequencies. From this figure one can also understand that for higher values of wave numbers, the dynamic gradient parameter has more effect. Furthermore, it is also seen that with the increase of static gradient parameter, the frequencies are increased. The figure indicates that one can easily find the nonlocal elasticity and gradient elasticity theory knowing that one parameter is not accurate in all cases. One of the important parameters which plays a major role in studying wave propagation is group velocity. The group velocity can be expressed as $\frac{d\omega}{dk}$. From Fig. 1, it is concluded that as the dynamic gradient parameter increase, the group velocities will decrease.

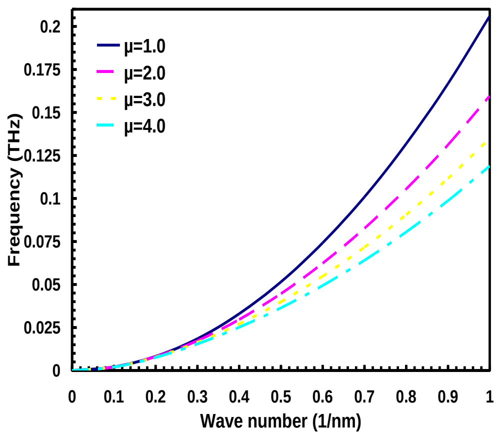
Figure 2 depicts the influences of magnetic field and dynamic gradient parameter on the frequencies of rectangular nanoplates. In this figure the parameter MP is defined as $MP = \frac{\eta h H_x^2 L^2}{D}$. It is shown that increasing the parameter MP will increase the frequencies. From this figure it can be found that the effects of magnetic field cannot be ignored. It seems that the dynamic gradient parameter has more effects when the parameter MP has lower value. It is mentioned that in this example the static gradient parameter is assumed to be 1.0 nm^2.



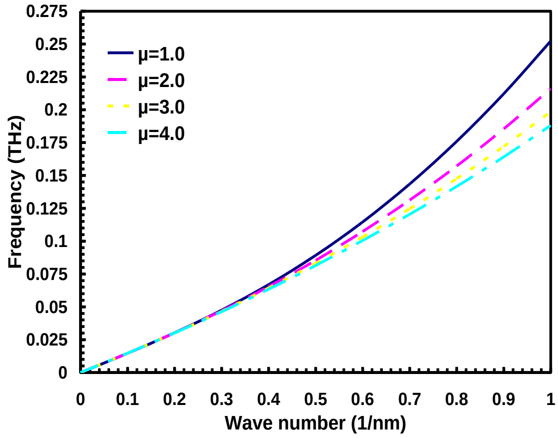
a) $\boldsymbol{l = 1.0}$ **b)** $\boldsymbol{l = 1.5}$

c) $\boldsymbol{l = 2.0}$

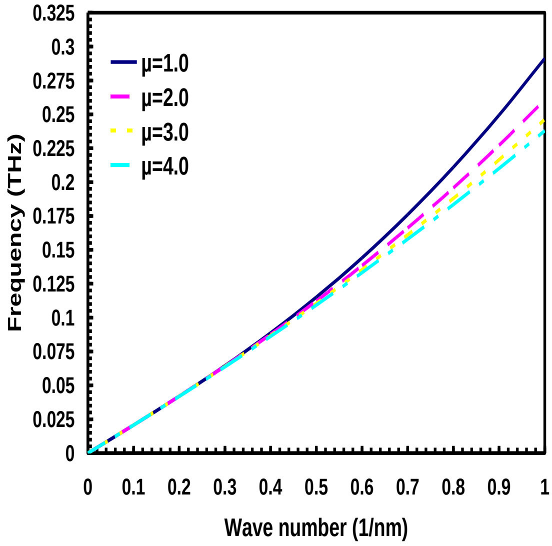
Fig. 1. The effects of static and dynamic gradient parameters on the frequencies of nanoplates with considering magnetic field (MP=25).



a) $\boldsymbol{MP = 0.0}$



b) $\boldsymbol{MP = 25}$

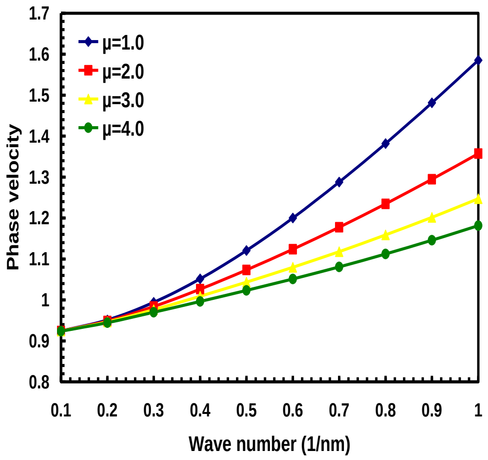


c) $\boldsymbol{MP = 50}$

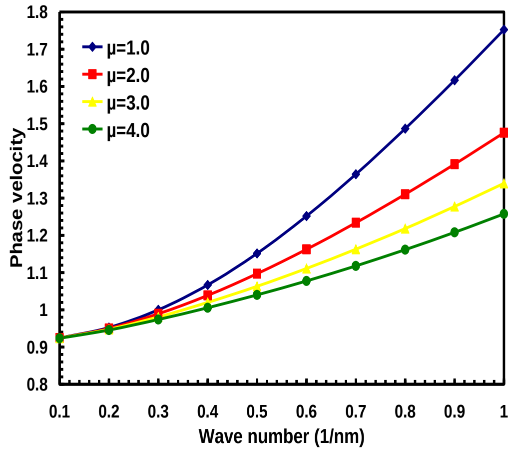
Fig. 2. The influences of magnetic field and dynamic gradient parameter on the frequencies ($l = 1.0nm^2$).

In Fig. 3, the influences of static and dynamic gradient parameters on the phase velocities are investigated. The phase velocity is defined as $\frac{\omega}{k}$ where k is the wave number. The figure illustrates that with the increase of dynamic gradient parameter, the phase velocities will decrease but increasing the static gradient parameter will increase the phase velocities. It is also depicted that increasing the wave numbers will cause increasing the phase velocities. Moreover, it can be concluded that classical plate theory for macro plate cannot be used for nanoplates and small scale effects should be considered.

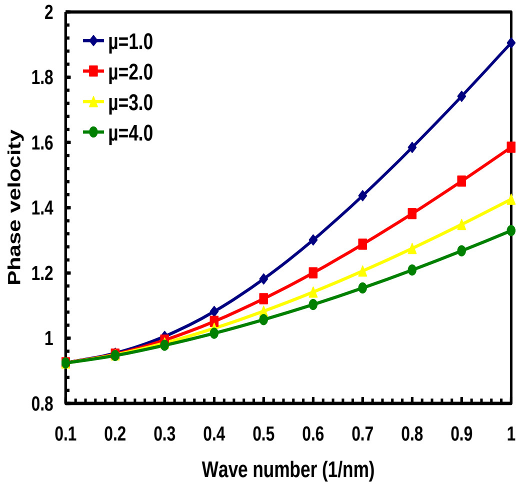
Figure 4 presents the variations of the phase velocities with respect to magnetic field and wave numbers. In this figure, the static and dynamic gradient parameters are assumed to be 1.0 nm^2 . It can be seen that the phase velocities increase for the increase of magnetic field. It is recommended to other researchers to use some other theories [16-18] in order to study wave propagation in rectangular nanoplates based on nonlocal elasticity theory and strain gradient elasticity theory.



a) $\boldsymbol{l = 1.0}$



b) $\boldsymbol{l = 1.5}$



c) $\boldsymbol{l = 2.0}$

Fig. 3. The effects of static and dynamic gradient parameters on the phase velocities of nanoplates considering magnetic field (MP=25).

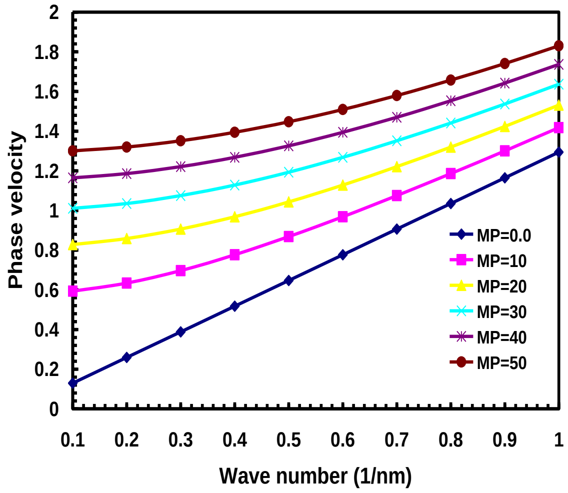


Fig. 4. The influences of magnetic field on the phase velocities ($l = 1.0nm^2 \mu = 1.0nm^2$).

5. Conclusion

In this paper, a closed form solution for the wave propagation in rectangular nanoplates based on Aifantis's strain gradient elasticity theory was proposed. The effects of in-plane magnetic field were investigated on the frequencies and phase velocities. It was shown that increasing the dynamic gradient parameter will decrease the frequencies. It was observed that with the increase of static gradient parameter, the frequencies have been increased. Moreover, the phase velocities increase for the increase of magnetic field.

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