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Genetic Algorithm-based Optimization Procedures to Find the Constants of Johnson-Cook Constitutive Equation

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ABSTRACT

Johnson-Cook constitutive equation is one of the most famous constitutive equations that have ever been developed to model the hot deformation flow curves of different materials. This equation is a predefined model in the traditional finite element codes to describe the material behavior in applications such as simulating the manufacturing processes. In this work, two different genetic algorithm-based (GA) optimization procedures, referred to as free and constrained optimization procedures, were proposed to find the constants of the Johnson-Cook constitutive equation. The proposed procedures were applied to fit the Johnson-Cook constitutive equation on the experimental flow curves of API X65 pipeline steel. According to the obtained constants, the modeling performances of the proposed procedures were compared with each other and with the modeling performance of the conventional procedure of finding the constants of the Johnson-Cook equation. Root mean square error (RMSE) criterion was used to asses and to compare the performances of the examined procedures. According to the obtained results, it was determined that the proposed free GA based optimization procedure with the RMSE value of 7.2 MPa had the best performance, while the performance of the conventional procedure was the worst.

1. Introduction

Johnson-Cook constitutive equation is one of the earliest constitutive equations that have ever been developed to model the response of different materials to external loading at elevated temperatures. This equation has been developed by Johnson and Cook and was used to model the flow curves of different metallic materials [1]. Until now, many other equations have been developed to model the hot deformation behavior of different metals and alloys. A critical review on the experimental results and constitutive descriptions for metals and alloys in hot working has been presented by Lin and Chen [2]. As presented by them, the constitutive models are divided into three categories, including the phenomenological, physical-based and artificial neural network models. According to this categorization, the Johnson-Cook constitutive equation is considered as a phenomenological model. In the Johnson-Cook model, the flow stress of the material is considered as the multiplication effects of strain, strain-rate, and temperature. The simplicity of this interpretation is the main advantage of the JC model; however, the coupling effects of strain, temperature and strain-rate have not been considered [2]. Therefore, some modifications have been proposed on this model to improve its modeling accuracy [3-5]. The original Johnson-Cook equation is not able to predict the softening part of the flow curves, and subsequently, the modifications developed based on the coupling effect of strain, strain

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rate and temperature are not able to account for the softening stage. This is the main shortcoming of this equation. To overcome this problem, the strain dependent term of the Johnson-Cook equation should be modified. Modification of the strain dependent term of the Johnson-Cook equation was proposed by Lin et al. [6], dividing the strain dependent term of the Johnson-Cook equation into two parts, one before and the other after the peak stress conducted by Akbari et al. [7]. These are some efforts made to overcome this problem. Consequently, more constitutive equations have been developed with a more precise prediction performance [8].

Despite the above-mentioned problems associated with applying the Johnson-Cook equation for the flow stress modeling, as this constitutive equation is a predefined model to describe the material behavior in the traditional finite element codes, developed to simulate the manufacturing processes of metallic materials [9, 10], many efforts have been made to find the constants of this equation for different materials [11, 12]. Usually, the results of the hot compression tests conducted at different deformation conditions (different temperatures and strain rates) for an interested metallic material are used to calibrate the constants of the Johnson-Cook constitutive equation regarding this material [7, 12].

Different strategies may be followed to obtain these constants that will affect the final modeling performance of the Johnson-Cook constitutive equation. Five different calibration strategies have been identified and discussed by Gambirasio and Rizzi [13]. However, as it is confirmed by them, these calibration strategies are not the only possible procedures [13].

Until now, genetic algorithm (GA) has been used in different fields of science and engineering [14-16]. It has been especially used as an optimization tool to fit a predefined equation on experimental data. For example, GA has been used to estimate some material properties, including Young's modulus, yield strength and hardening modulus in terms of temperature and strain rate [15].

In this work, two different genetic algorithm-based optimization procedures were proposed to find the constants of the Johnson-Cook constitutive equation. The proposed procedures were applied to fit the Johnson-Cook constitutive equation on the experimental flow curves of API X65 pipeline steel (conducted at different temperatures and strain rates) as a case study. According to the obtained constants, the modeling performances of the proposed procedures were compared with each other and with the modeling performance of the conventional procedure of finding the constants of the Johnson-Cook constitutive equation. Root mean square error (RMSE) criterion was used for evaluating and comparing the performances of the examined procedures.

2. Experimental Flow Curves of API X65

The steel, used in the test, is a high-strength low alloy (HSLA) steel that is produced by thermo-mechanical controlled rolling and is used in the construction of large-diameter gas pipelines [17]. The chemical and mechanical specifications of this steel are characterized by API standard code [18].

Single hit compression tests were conducted on cylindrical specimens being 10 mm in diameter and 15 mm in length. They were machined to the original pipe with the longitudinal axis parallel to the rolling direction. All tests were carried out on a 250 kN Zwick tensile/compression testing machine equipped with a radiant furnace with the temperature accuracy of ±5°C [17]. The results of the hot compression tests which were conducted at temperatures of 950, 1000, 1050, 1100 and 1150°C with different strain rates of 0.01, 0.1 and 1 s^{-1} for each of the deformation temperatures under true strain of about 0.7 are presented in Fig. 1 [17].

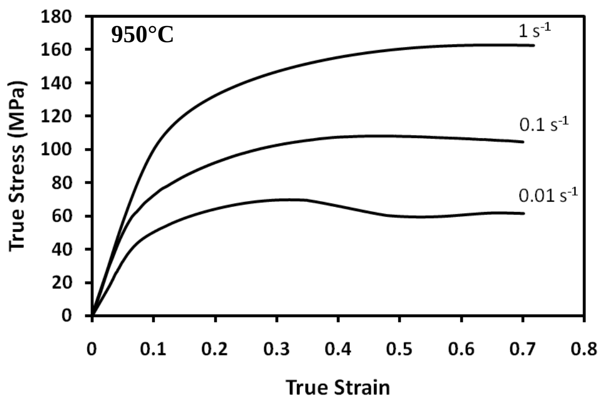


Fig. 1. Experimental flow curves of API X65 pipeline steel at different temperatures and strain rates [17].

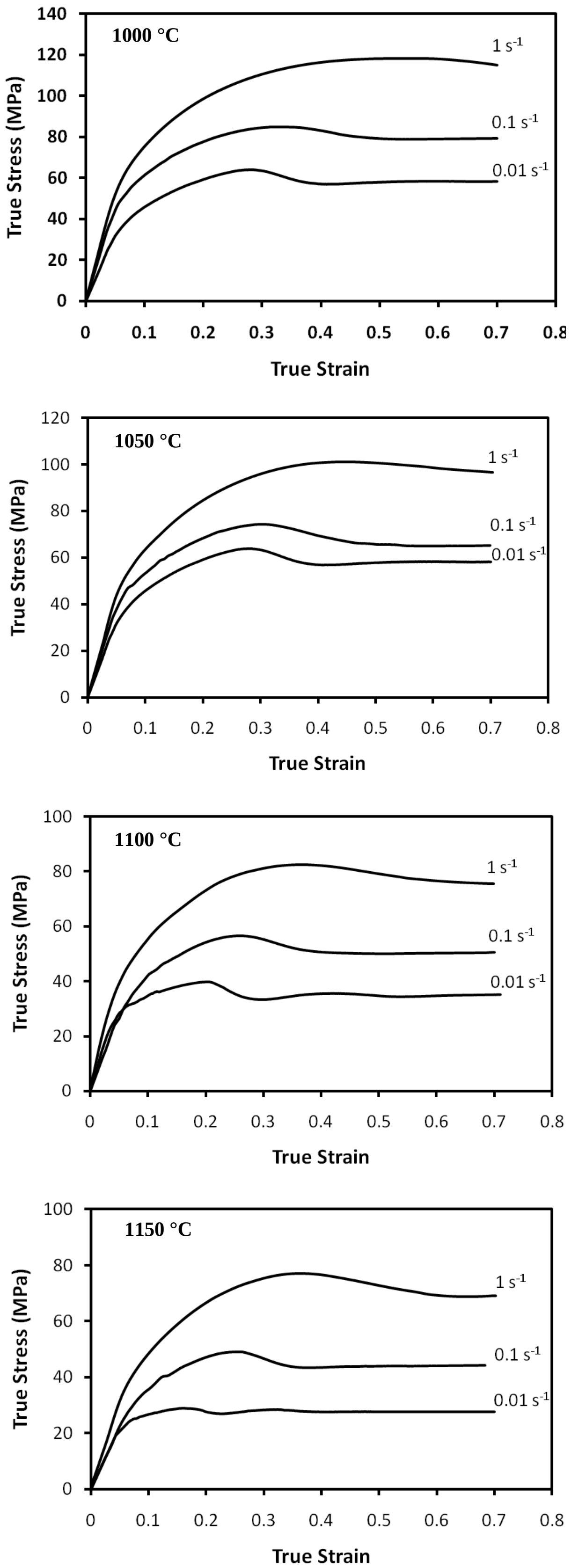


Fig. 1. Continue

3. The Results of Different Examined Procedures to Find the Constants of the Johnson–Cook Constitutive Model

In this section, the conventional procedure of finding the constants of the Johnson-Cook constitutive equation together with the two proposed genetic algorithm-based optimization procedures are described. The conventional procedure of finding the constants of the Johnson-Cook constitutive equation can be found in the work of He et al. [19], Abbasi-Bani et al. [20] and Akbari et al. [7] for modeling the flow curves of 20CrMo alloy steel, Mg–6Al–1Zn alloy and medium carbon microalloyed steel, respectively. The examined procedures were applied to fit the Johnson-Cook constitutive model on the experimental flow curves of API X65 pipeline steel. Then, the RMSE criterion was used to evaluate and compare the performance of the examined procedures.

3.1. Conventional procedure of finding the constants of the Johnson-Cook constitutive equation

As suggested by Johnson and Cook [1], considering the effects of the strain, strain rate and deformation temperature, the following constitutive equation can be used to describe the flow stress of different materials:

$$\sigma = (\sigma_{yr} + B\varepsilon^p)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)\left(1 - \left(\frac{T-T_r}{T_m-T_r}\right)^q\right) \quad (1)$$

Where ε is the strain, $\dot{\varepsilon}$ is the strain rate, $\dot{\varepsilon}_r$ is the reference strain rate, T is the absolute deformation temperature, T_r is the reference deformation temperature, T_m is the melting temperature of the material (1500ºC (1773 K) for the tested steel), σ_{yr} is the material's yield strength at the reference strain rate and reference temperature condition and B, C, p and q are the material's constants. In the Johnson-Cook constitutive equation, the first bracket is applied to describe the effect of strain hardening and the second and third brackets are used to compensate the effects of the strain rate and the temperature on the flow curves, respectively [7, 19, 20].

According to the literature review [7, 19, 20], the following conventional procedure is usually applied to obtain the constants of the Johnson-Cook constitutive equation:

1) The lowest examined strain rate or the strain rate of 1 s^{-1} is considered as the reference strain rate. Furthermore, the lowest examined temperature is considered as the reference temperature. At the deformation condition with the reference strain rate and the reference temperature, the second and third brackets equal to 1; so, Eq. 1 is simplified to Eq. 2 under this condition:

$$\sigma = \sigma_{yr} + B\varepsilon^{p} \quad (2)$$

Taking the natural logarithm from the above equation, gives:

$$ln(\sigma - \sigma_{yr}) = lnB + p \ln \varepsilon \quad (3)$$

Therefore, the plot of ln $(\sigma - \sigma_{yr})$ versus ln ε is used to obtain the values of B and p (σ_{yr} can be obtained from the experimental flow curve at the reference condition).

2) Writing Eq. 1 for the reference temperature conditions yields:

$$\sigma = (\sigma_{yr} + B\varepsilon^{p})\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right) \quad (4)$$

As a result, the plot of $\sigma/(\sigma_{yr} + B\varepsilon^{p})$ versus $\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}$ can be used to calculate the average value of C.

3) Writing Eq. 1 for the reference strain rate conditions yields:

$$\sigma = (\sigma_{yr} + B\varepsilon^{p})\left(1 - \left(\frac{T-T_r}{T_m-T_r}\right)^{q}\right) \quad (5)$$

Taking the natural logarithm from the above equation, results in:

$$\ln\left[1 - \left(\frac{\sigma}{(\sigma_{yr}+B\varepsilon^{p})}\right)\right] = q\, ln\left(\frac{T-T_r}{T_m-T_r}\right) \quad (6)$$

So, the plot of $ln\left[1 - \left(\frac{\sigma}{(\sigma_{yr}+B\varepsilon^{p})}\right)\right]$ versus $ln\left(\frac{T-T_r}{T_m-T_r}\right)$ can be used to obtain the value of q.

Here, the above-mentioned conventional procedure was applied to obtain the constants of the Johnson-Cook constitutive equation for the tested steel:

1) The deformation condition with the strain rate of 1 s^{-1} and the lowest temperature (the temperature of 950 °C (1223 K) was considered as the reference deformation condition. Consequently, the plot of $ln(\sigma - \sigma_{yr})$ versus $\ln \varepsilon$ was used for the values of B and p (σ_{yr} was obtained from the experimental flow curve at the reference condition which equals to 56.47 MPa). The plot of $ln(\sigma - \sigma_{yr})$ versus $\ln \varepsilon$ is presented in Fig. 2.

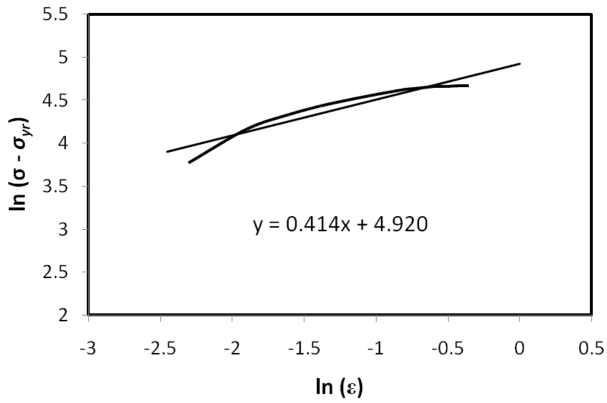


Fig. 2. The plot of $ln(\sigma - \sigma_{yr})$ vs. $\ln \varepsilon$ to obtain the values of B and p.

As can be observed in this figure, the best fitted line was used to calculate the values of lnB and p. Accordingly, the values of B and p were obtained as 137.00 and 0.414, respectively.

2) The plot of $\sigma/(\sigma_{yr} + B\varepsilon^{p})$ versus $\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}$ was used to calculate the average value of C (see Fig. 3). It should be mentioned that this is the average value of C constants, obtained at different strains in the range of 0.05 to 0.7 and with the step size of 0.05.

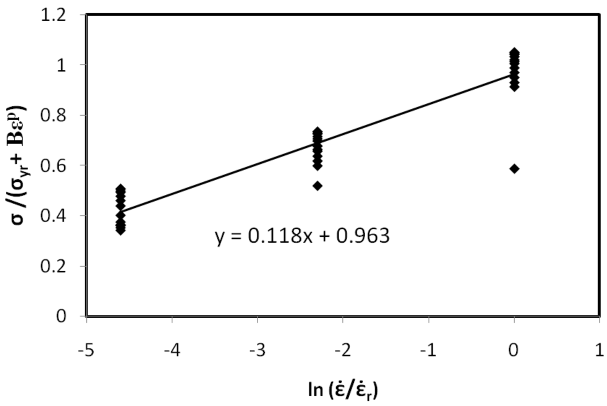


Fig. 3. The plot of $\sigma/(\sigma_{yr} + B\varepsilon^{p})$ vs. $ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}$ to calculate the value of C.

As can be seen in this figure, using the linear fitting, the average value of C was obtained as 0.118.

3) The plot of $ln\left[1 - \left(\frac{\sigma}{(\sigma_{yr}+B\varepsilon^{p})}\right)\right]$ versus $ln\left(\frac{T-T_r}{T_m-T_r}\right)$ was used to obtain the average value of q (Fig. 4). It should be mentioned that this is the average value of q constant, obtained at different strains in the range of 0.05 to 0.7 and with the step size of 0.05.

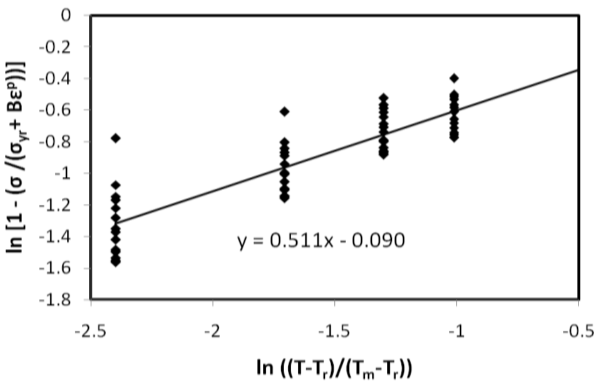
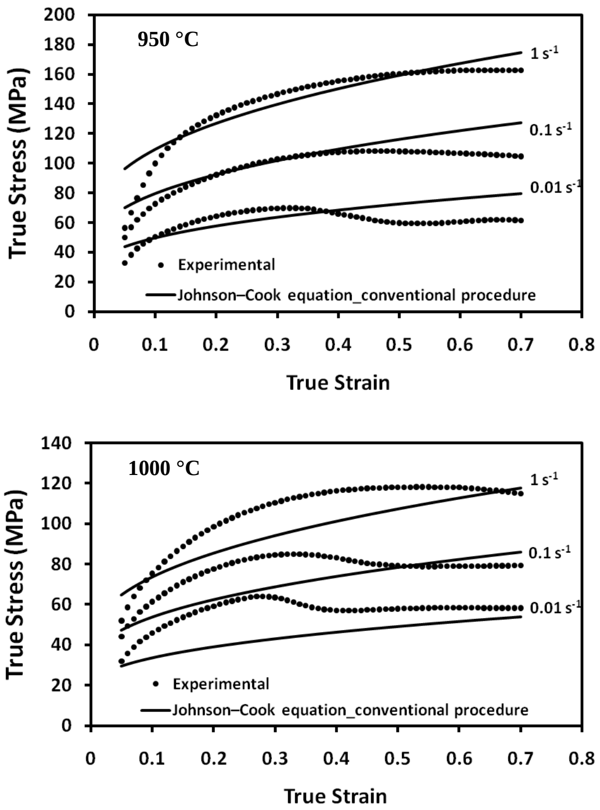


Fig. 4. The plot of $ln\left[1-\left(\frac{\sigma}{(\sigma_{yr}+B\varepsilon^{p})}\right)\right]$ vs. $ln\left(\frac{T-T_r}{T_m-T_r}\right)$ to obtain the value of q.

As can be seen, using the linear fitting, the average value of q was obtained as 0.511. Substituting the obtained constants, the Johnson-Cook constitutive equation was rewritten for the tested steel as follows:

$$\sigma=(56.47+137.00\varepsilon^{0.414})(1+0.118\ln\dot{\varepsilon})\left(1-\left(\frac{T-1223}{1773-1223}\right)^{0.511}\right) \quad (7)$$

A comparison between the experimental and modeled flow curves of the tested steel (using the Johnson-Cook constitutive equation with the constants obtained from the conventional procedure) at different deformation conditions is presented in Fig. 5.



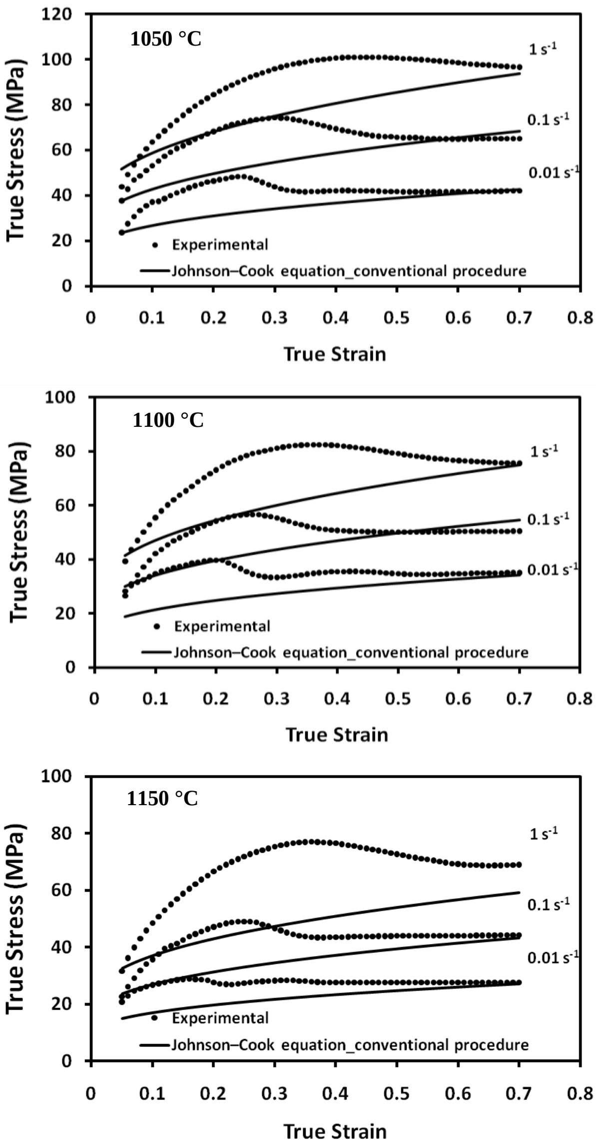


Fig. 5. The comparison between the experimental and modeled flow curves (using the Johnson-Cook constitutive equation with the constants obtained from the conventional procedure) at different hot deformation conditions for API X65 pipeline steel.

As depicted in Fig. 5, though the softening stage of the flow curves of the tested steel cannot be modeled using the Johnson-Cook constitutive equation, an acceptable prediction performance can be observed at lower examined temperatures. However, at higher examined temperatures (1100 and 1150°C), the performance of the developed Johnson-Cook model is not reliable. The modeling performance of this procedure of finding the constants of the Johnson-Cook constitutive equation will be quantitatively assessed in section 3.3 of the manuscript.

3.2. GA based procedures to find the constants of the Johnson-Cook constitutive model

GA is an optimization procedure that has been developed based on Darwin's survival of the fittest principles. In GA, a population of possible solutions (called chromosomes) is used to solve a problem. The performances of each chromosome are evaluated through a fitness function. Then, the next generation of the solutions can be produced by the operators of selection, crossover and mutation. The process of producing and evaluating the generations is continued while a stopping criterion is reached. GA will not always find the exact optimum solution but will typically find a solution very close to the optimum [21]. Here, two different genetic algorithm-based optimization procedures were proposed to find the constants of the Johnson-Cook constitutive equation. The principles of these genetic algorithm-based optimization procedures are explained as follows:

The first proposed GA based optimization procedure that is referred to as the constrained GA based optimization procedure (in this paper) can be conducted through the following two steps:

1) The constants of B and p can be determined by writing the Johnson-Cook equation for the deformation condition with the reference strain rate and the reference temperature (similar to what was conducted in the first step of the conventional procedure).

2) Then, the GA based optimization procedure is applied to adjust the other constants of the Johnson-Cook equation so as to minimize the sum of squared errors between the modeled and experimental flow curves of an interested material. This means that the following equation should be fitted on the experimental flow curves of API X65 steel (as a case study):

$$\sigma = (56.47 + 137.00\varepsilon^{0.414})\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)\left(1 - \left(\frac{T-950}{1500-950}\right)^q\right) \quad (8)$$

Note that in the above equation, the constants of B and p, obtained from the first step, together with the value of the yield strength, obtained from the stress-strain flow curve at the reference deformation condition, are introduced in the Johnson-Cook equation.

The second proposed GA based optimization procedure that is referred to as the GA based optimization procedure (in this paper) is applied freely to adjust the six constants of the Johnson–Cook equation so as to minimize the sum of squared errors between the modeled and experimental flow curves of an interested material. The six constants of the Johnson-Cook equation includes σ_{yr}, B, p, C, q and $\dot{\varepsilon}_r$. Since the melting temperature is one of the intrinsic properties of the material and selecting the lowest examined temperature as the reference temperature to avoid the negative value for the homogeneous temperature is a common practice [13], these two constants of the Johnson-Cook equation were ignored in this proposed optimization procedure. This means that the following equation should be fitted on the experimental flow curves of API X65 steel (as a case study):

$$\sigma = (\sigma_{yr} + B\varepsilon^p)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^q\right) \quad (9)$$

The implemented details of both constrained and free GA based optimization procedures are presented in the rest of the paper.

3.2.1. Fitness function for the proposed optimization procedures

The main idea of the proposed optimization procedures, here, is to adjust some of the constants of the Johnson-Cook equation so as to minimize the deviation between the experimental and modeled flow stresses. Since the deviation between the experimental and modeled flow stresses can be quantified using the sum of squared errors, the following fitness function (Eq. 10) was used in this work:

$$Fitness\ Function = \sum_{i=1}^{N}\left(\sigma_i^{exp.} - \sigma_i^{model}\right)^2 \quad (10)$$

Where $\sigma_i^{exp.}$ is the experimental flow stress; σ_i^{model} is the modeled flow stress and N is the number of measured points. Substituting Eq. 8 with the original fitness function (Eq. 10) yields Eq. 11 as the fitness function, for the constrained GA based optimization procedure:

$$Fitness\ Function = \sum_{i=1}^{N}\left[\sigma_i^{exp.} - \left[(56.47 + 137.00\varepsilon^{0.414})\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^q\right)\right]\right]^2 \quad (11)$$

In a similar way, substituting Eq. 9 with the original fitness function (Eq. 10) yields Eq. 12 as the fitness function for the free GA based optimization procedure:

$$Fitness\ Function = \sum_{i=1}^{N}\left[\sigma_i^{exp.} - \left[(\sigma_{yr} + B\varepsilon^p)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_r}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^q\right)\right]\right]^2 \quad (12)$$

The fitness functions together with the experimental flow curves were fed to MATLAB genetic algorithm toolbox to execute the proposed procedures. The parameters used to run the MATLAB GA toolbox are introduced in the next section.

3.2.2. Initialization, selection, crossover and mutation

The initial rage of the problem parameters should be selected correctly. Diversity of the population and consequently the performance of the genetic algorithm are affected by the initial rage of the problem parameters [21]. With regard to the literature survey [2, 7, 11, 12, 19, 22-24], the initial range of the six constants of the Johnson-Cook equation, including σ_{yr}, B, p, C, q and $\dot{\varepsilon}_r$ for different steel types are presented in Table 1.

Table 1. The constants of Johnson-Cook constitutive equation for some different steel types

Steel type/ Reference	σ_{yr}	B	p	C	q	$\dot{\varepsilon}_r$
Austenitic Stainless Steel 316 (Gupta et al. 2013)	265.45	1504.3	0.7954	0.0061	0.7623	1
a medium carbon microalloyed steel (Akbari et al. 2015)	21.7	36.67	0.128	0.221	0.444	0.0001
20CrMo alloy steel (He et al. 2013)	27.842	97.014	0.56	0.15	0.89	0.005
A typical high-strength alloy steel (Lin and Chen, 2010)	102.6	80.18	0.5611	0.11096	0.6874	1
AISI1006 steel (Wang, 2006)	350	275	0.36	1	0.022	1
AISI4340 steel (Wang, 2006)	792	509	0.26	1	0.014	1
S7 tool steel (Wang, 2006)	1539	476	0.18	1	0.012	1
304 Stainless steel (Dean et al. 2011)	310	1000	0.65	1	0.07	0.01
titanium-modified austenitic stainless steel (Samantaray et al. 2009)	120	465.79	0.308	0.1	0.75	1
A typical high-strength alloy steel (Lin et al. 2010)	35.49	76.79	0.5922	0.1752	0.6259	0.0001

Accordingly, the lower and upper bounds of these constants of the Johnson-Cook equation are summarized in Table 2.

Table 2. The lower and upper bounds of the constants of Johnson-Cook for some different steel types

	σ_{yr}	B	p	C	q	$\dot{\varepsilon}_r$
Upper bound	1539	1504.3	0.7954	1	0.89	1
Lower bound	21.7	36.67	0.128	0.0061	0.012	0.0001

These lower and upper bounds were applied for both constrained and free proposed optimization procedures.

In GA, some of the best fitted solutions (chromosomes) called elite children are chosen to participate in the next generation. Further, some of the chromosomes, excluded from the elite children, are selected to be the parents of the next generation by a selecting process. The selecting process gives a higher chance to the best fitted chromosomes to be chosen and also allows the less fitted chromosomes to be selected for the sake of maintaining the diversity of the next generation. A fraction of these parents generates the children of the next generation by the crossover operator, while others generate some children by the mutation operator. Different crossover ratios can be examined; however, the typical range of it is from 0.5 to 0.8 [25].

Here, to run the MATLAB genetic algorithm toolbox for the implementation of the proposed constrained GA based optimization procedure, the population size was set as 20 and the number of the elite children was set as 2. Stochastic uniform process was used to select the parents of the next generation. Moreover, the crossover ratio was set as 0.8 and the process was stopped when the average change in the fitness function value over 10 stall generations was less than 1 e-6. The plot of the best fitness together with the mean fitness against the generation number is presented in Fig. 6 (for the proposed constrained GA based optimization procedure).

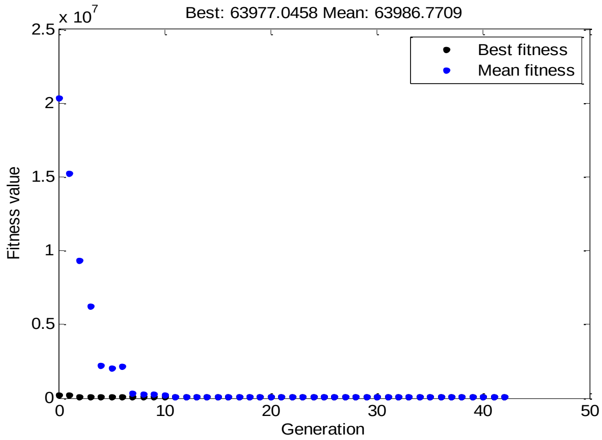


Fig. 6. The plot of the best and mean fitness values obtained at each generation (for the proposed constrained GA based optimization procedure).

As it can be observed in Fig. 6, in the initial generations, the values of the mean fitness are very different from the values of the best fitness. On the other hand, in the final generations, the mean fitness values move toward the best fitness values. After implementing the constrained GA based optimization procedure, the values of 0.122 and 0.570 returned for the constants of C and q, respectively. The overall results are presented in the next section.

On the other hand, in executing the free GA based optimization procedure, the population size, the number of the elite children and the crossover ratio were set as 60, 6 and 0.8, respectively. The method converged when the average change in the fitness function value over 10 stall generations was less than 1 e-9. The plot of the best fitness together with the mean fitness against the generation number is presented in Fig. 7 (for the proposed free GA based optimization procedure).

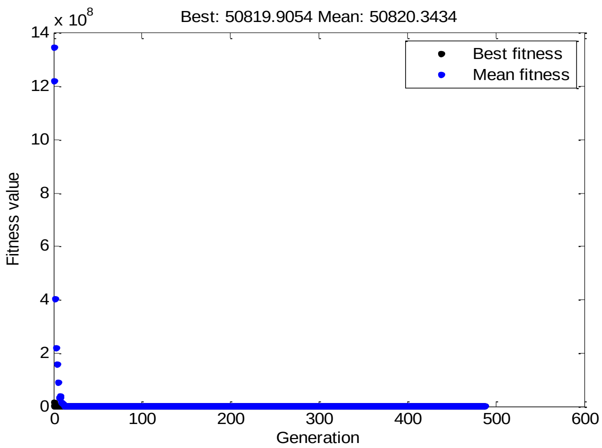


Fig. 7. The plot of the best and mean fitness values obtained at each generation (for the proposed free GA based optimization procedure).

As it can be observed in Fig. 7, by ascending the generations, the mean fitness values move toward the best fitness values. After implementing the free GA based optimization procedure, the values of 23.75, 109.09, 0.233, 0.156, 0.594 and 0.1415 returned for the constants of σ_{yr}, B, p, C, q and $\dot{\varepsilon}_r$, respectively. The overall results are presented in the next section.

3.2.3. Results of the GA based optimization procedures

Fitting the returned values of 0.122 and 0.570 for the constants of C and q, (obtained from implementing the constrained GA based optimization procedure) into Eq. 8, the following rewritten Johnson-Cook equation can be used to describe the flow curves of API X65 pipeline steel:

$$\sigma = (56.47 + 137.00\varepsilon^{0.414})\left(1 + 0.122\ \ln\frac{\dot{\varepsilon}}{1}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^{0.570}\right) \qquad (13)$$

A comparison between the experimental and modeled flow curves of the tested steel (using the constrained GA based optimization procedure) at different deformation conditions is presented in Fig. 8.

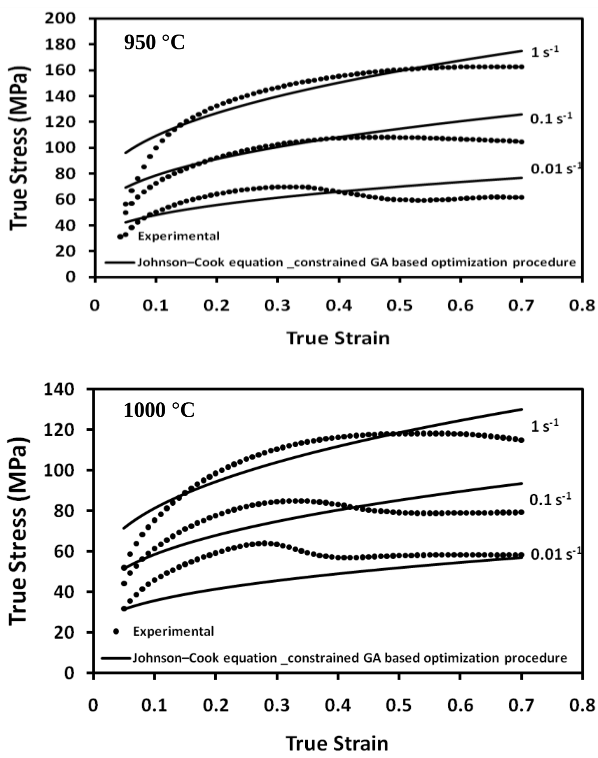


Fig. 8. The comparison between the experimental and modeled flow curves (using the Johnson-Cook constitutive equation with the constants obtained from the proposed constrained GA based optimization procedure) at different hot deformation conditions for API X65 pipeline steel.

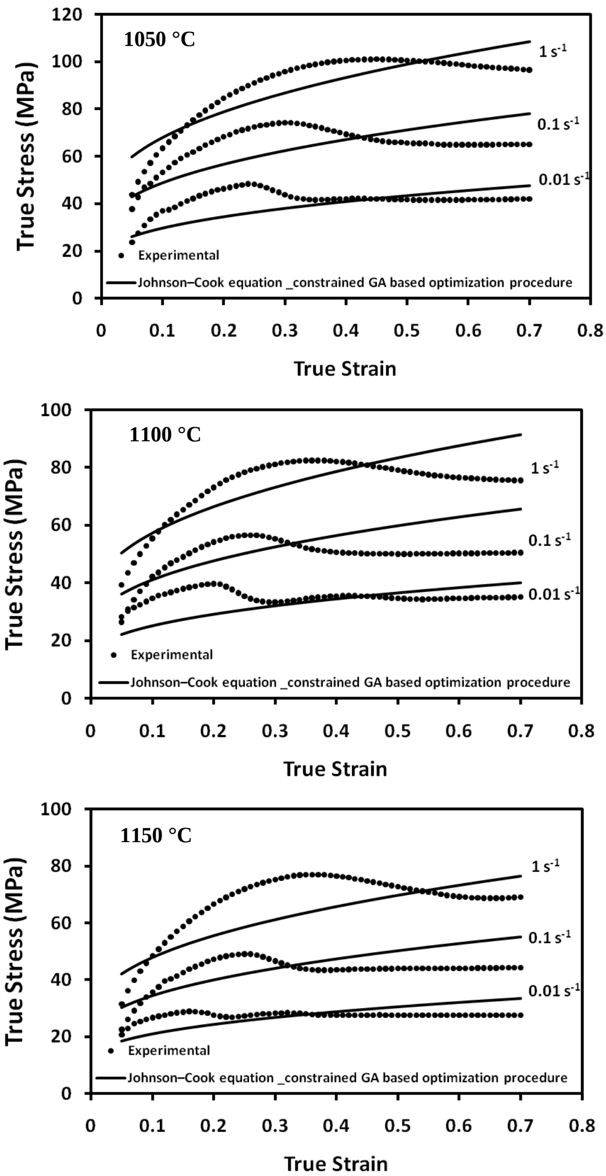


Fig. 8. Continue

Fitting the returned values of 23.75, 109.09, 0.233, 0.156, 0.594 and 0.1415 for the constants of σ_{yr}, B, p, C, q and $\dot{\varepsilon}_r$, (obtained from implementing the free GA based optimization procedure) into Eq. 9, the following rewritten Johnson-Cook equation can be used to describe the flow curves of API X65 pipeline steel:

$$\sigma = (23.75 + 109.09\varepsilon^{0.233})\left(1 + 0.156\ \ln\frac{\dot{\varepsilon}}{0.1415}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^{0.594}\right) \quad (14)$$

A comparison between the experimental and modeled flow curves of the tested steel (using the free GA based optimization procedure) at different deformation conditions is presented in Fig. 9.

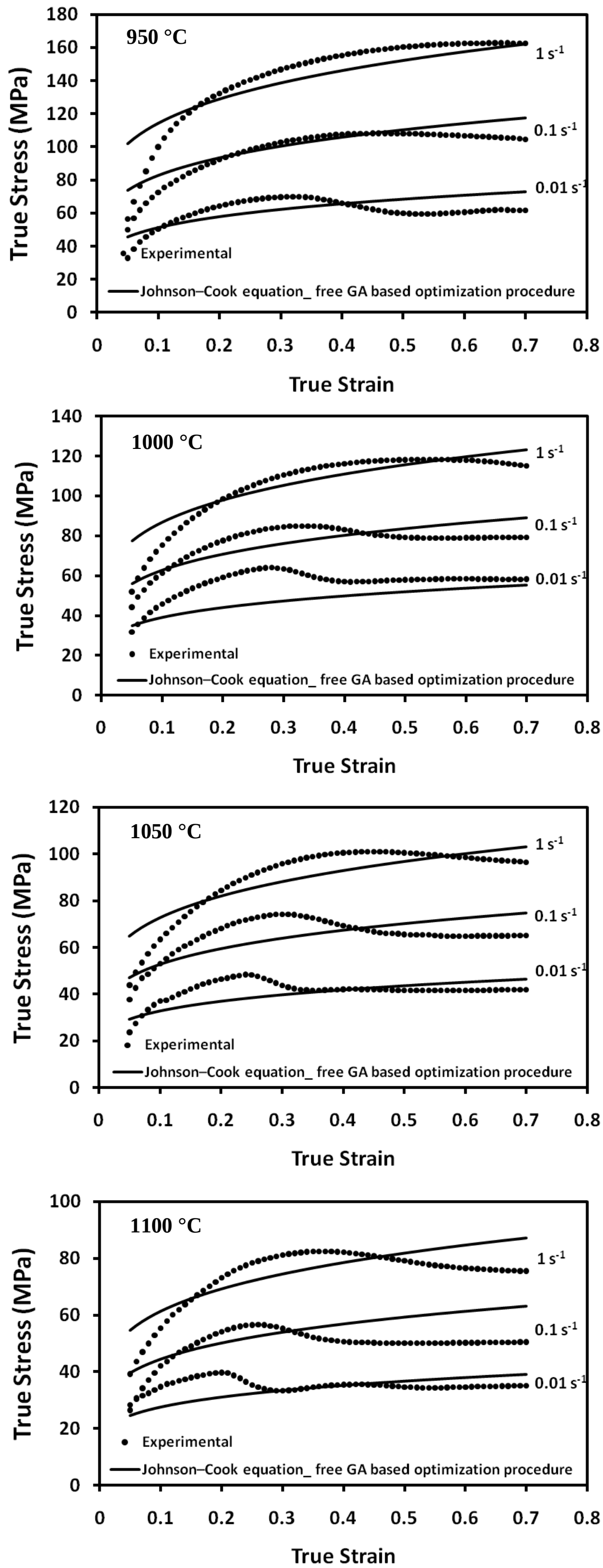


Fig. 9. The comparison between the experimental and modeled flow curves (using the Johnson−Cook constitutive equation with the constants obtained from the proposed free GA based optimization procedure) at different hot deformation conditions for API X65 pipeline steel.

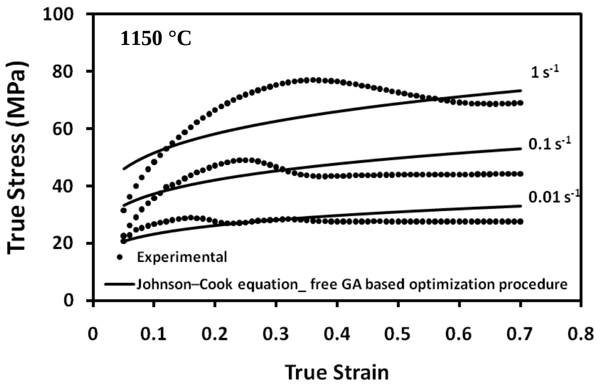


Fig. 9. Continue

3.3. Comparison of the results

In this section, the results of the proposed GA based optimization procedures for finding the constants of the Johnson-Cook constitutive equation are compared with each other and with the results of the conventional procedure. The root mean square error (RMSE) criterion was used for this purpose:

$$\mathrm{RMSE} = \frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\left(\sigma_i^{exp.} - \sigma_i^{model}\right)^2 \qquad (15)$$

Where $\sigma_i^{exp.}$, σ_i^{model} and N are the same as those defined in Eq. 10. The RMSE values obtained for the fifteen flow curves of API X65 pipeline steel (Fig. 1) by the examined procedures to find the constants of the Johnson-Cook constitutive equation are presented in Table 3.

Table 3. Root Mean Square Error (MPa) between the experimental and modeled flow curves of tested steel using the different examined procedures

Examined procedures of finding the constants of Johnson-Cook equation	RMSE (MPa)
Conventional procedure	11.53
Proposed Constrained GA based procedure	8.04
Proposed Free GA based procedure	7.16

As presented in Table 3, the Proposed Free GA based procedure to find the constants of the Johnson-Cook equation has the best performance. Therefore, this optimization procedure is suggested to determine the constants of the Johnson-Cook equation for different materials.

4. Conclusions

In this work two different genetic algorithms, (GA) based optimization procedures (referred to as free and constrained optimization procedures), were proposed to find the constants of the Johnson-Cook constitutive equation. The proposed procedures were applied to fit the Johnson-Cook constitutive equation on the experimental flow curves of API X65 pipeline steel. According to the constants obtained, the modeling performances of the proposed procedures were compared with each other and with the modeling performance of the conventional procedure of finding the constants of the Johnson-Cook equation. The overall results can be summarized as follows:

1- According to the constants of the Johnson-Cook equation obtained by using the conventional procedure, the flow stress of the tested steel can be described by the following rewritten Johnson-Cook equation:

$$\sigma = (56.47 + 137.00\varepsilon^{0.414})(1 + 0.118\ln\dot{\varepsilon})\left(1 - \left(\frac{T-1223}{1773-1223}\right)^{0.511}\right)$$

2- According to the constants of the Johnson-Cook equation obtained by using the proposed constrained GA based procedure, the flow stress of the tested steel can be described by the following rewritten Johnson-Cook equation:

$$\sigma = (56.47 + 137.00\varepsilon^{0.414})\left(1 + 0.122\ \ln\frac{\dot{\varepsilon}}{1}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^{0.570}\right)$$

3- According to the obtained constants of the Johnson-Cook equation using the proposed free GA based procedure, the flow stress of the tested steel can be described by the following rewritten Johnson-Cook equation:

$$\sigma = (23.75 + 109.09\varepsilon^{0.233})\left(1 + 0.156\ \ln\frac{\dot{\varepsilon}}{0.1415}\right)\left(1 - \left(\frac{T-1223}{1773-1223}\right)^{0.594}\right)$$

4- Root mean square error (RMSE) criterion was applied to compare the performances of the examined procedures. RMSE values of 11.53, 8.04 and 7.16 MPa were obtained for the conventional procedure, proposed constrained GA based procedure and proposed free GA based procedure, respectively. As the final result, it was concluded that the proposed free GA based procedure has the best performance among the examined procedures.

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استفاده از روش‌های بهینه‌یابی مبتنی بر الگوریتم ژنتیک جهت پیدا کردن ثابت‌های معادله جامع جانسون-کوک

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چکیـــده

معادله جامع جانسون-کوک یکی از مشهورترین معادلات جامعی است که تاکنون برای مدل کردن منحنی‌های سیلان مواد مختلف توسعه داده شده است. این معادله یکی از مدل‌های پیش‌فرض در نرم‌افزارهای تجاری اجزاء محدود برای توصیف رفتار مواد در کاربردهایی نظیر شبیه‌سازی فرآیندهای تولید است. در این مطالعه، دو روش بهینه‌یابی مبتنی بر الگوریتم ژنتیک متفاوت آزاد و مقید برای پیدا کردن ثابت‌های معادله جامع جانسون-کوک پیشنهاد شده است. از روش‌های پیشنهادی برای انطباق معادله جامع جانسون-کوک بر منحنی‌های سیلان تجربی فولاد خط لوله API X65 استفاده شده است. بر مبنای ثابت‌های به‌دست آمده، عملکرد روش‌های پیشنهادی با یکدیگر و با روش معمولی به‌دست آوردن ثابت‌های معادله جانسون-کوک مقایسه شد. از معیار ریشه میانگین مربعات خطا برای ارزیابی و مقایسه عملکرد روش‌های مورد مطالعه استفاده شد. با توجه به نتایج به‌دست آمده معین گردید که روش پیشنهادی بهینه‌یابی مبتنی بر الگوریتم ژنتیک آزاد با ریشه میانگین مربعات خطای برابر MPa ۷/۲ بهترین عملکرد را دارد؛ در حالی که، روش معمولی تعیین ثابت‌های معادله جانسون-کوک بدترین عملکرد را دارد.

واژه‌های کلیدی: الگوریتم ژنتیک، تغییر شکل گرم، تنش سیلان، معادلات جامع، معادله جانسون-کوک، فولاد خط لوله API